

Exercise 46

Find the derivative of the function.

$$y = [x + (x + \sin^2 x)^3]^4$$

Solution

Take the derivative using the chain rule.

$$\begin{aligned} y' &= \frac{dy}{dx} = \frac{d}{dx} [x + (x + \sin^2 x)^3]^4 \\ &= 4 [x + (x + \sin^2 x)^3]^3 \cdot \frac{d}{dx} [x + (x + \sin^2 x)^3] \\ &= 4 [x + (x + \sin^2 x)^3]^3 \cdot \left[1 + 3(x + \sin^2 x)^2 \cdot \frac{d}{dx} (x + \sin^2 x) \right] \\ &= 4 [x + (x + \sin^2 x)^3]^3 \cdot \left[1 + 3(x + \sin^2 x)^2 \cdot \left(1 + 2 \sin x \cdot \frac{d}{dx} \sin x \right) \right] \\ &= 4 [x + (x + \sin^2 x)^3]^3 \cdot [1 + 3(x + \sin^2 x)^2 \cdot (1 + 2 \sin x \cdot \cos x)] \\ &= 4 [x + (x + \sin^2 x)^3]^3 \cdot [1 + 3(x + \sin^2 x)^2 \cdot (1 + \sin 2x)] \end{aligned}$$